Physics I
ISI B.Math
Final Exam : April 25, 2016

Total Marks: 50
Time : 3 hours

Answer all the questions.

1. (**Marks = 5 × 2**)

Choose the correct option

i) Which of the following statements is not true in general about a closed system of N particles whose total linear momentum is conserved.

a) The Lagrangian of the system is invariant under spatial translations of the entire system .
b) The mutual internal force between any two particles of the system must act along the straight line joining the two particles.
c) The centre of mass of the system moves with uniform velocity.
d) The motion of the centre of mass is independent of the specific nature of the internal forces.

ii) The number of degrees of freedom of four rigid rods flexibly jointed to form a quadrilateral which can slide on a flat table is

a) 6
b) 2
c) 4
d) 3

iii) A particle of mass m is moving under the influence of a force $\mathbf{F} = -k\mathbf{r}$. Which of the following statements is false ?

a) Angular momentum about the origin is conserved.
b) Total energy is conserved
c) The orbit of the particle is a hyperbola
d) The orbit of the particle is an ellipse

iv) Which of the following Lagrangians does not correctly describe the one dimensional motion of a mass m attached to a spring of spring constant k ?

a) $L = \frac{1}{2}m\dot{x}^2 - \sqrt{mk}x\dot{x} - \frac{1}{2}kx^2$
b) $L = \frac{1}{2}m\dot{x}^2 + \alpha\dot{x} - \frac{1}{2}kx^2$ (where α is a constant)
c) $L = \frac{1}{2}m\dot{x}^2 + \beta x\dot{x}^2 - \frac{1}{2}kx^2$ (where β is a constant)
d) $L = \frac{1}{2}m\dot{x}^2 + \gamma x^2\dot{x} - \frac{1}{2}kx^2$ (where γ is a constant)

v) A uniform ball of mass M and radius a is pivoted so that it can turn freely about one of its diameters which is fixed in a vertical position. A beetle of mass m can crawl on the surface of the ball. Initially the ball is rotating with angular speed Ω with the beetle at the North pole. The beetle walks to the equator and stops. The final angular velocity Ω' of the ball is

(a) equal to Ω
(b) greater than Ω
(c) less than Ω
(d) less than or greater than Ω depending on the path taken by the beetle to reach the equator.

2. (**Marks** $= \mathbf{3+2+2+3}$)

A particle P of mass m is free to slide on a frictionless table and is connected via a string of length l that passes through a hole O in the table to a mass M that hangs below. Assume that M moves in vertical line only. Suppose that the length of the part of the string on the table is $R(t)$ at a time t and $\theta(t)$ is the angle between OP and and some fixed reference line on the table that passes through O .

(a) Write down the Lagrangian of the system in terms of the generalized coordinates (R, θ) and hence find the equations of motion for the system.

(b) Identify the cyclic coordinate and find the corresponding conserved generalized momentum. What is the symmetry of the Lagrangian that is associated with this conserved quantity ?

(c) If the mass m undergoes circular motion, find the radius R_0 of the circular orbit in terms of the conserved generalized momentum.

(d) Find the frequency of small radial oscillations about this circular orbit.

3. (**Marks** $= \mathbf{3+2+5}$)

(a) Show that $T = T^{CM} + T^G$ where T is the total kinetic energy of a system of particles , T^{CM} is the kinetic energy of the centre-of mass (given by $\frac{1}{2}MV^2$, where M is the sum of the mass of all the particles and V is the speed of the centre of mass) and T^G is the total kinetic energy of motion of the particles relative to the centre of mass.

(b) Using the result in (a), show that if a collision is elastic in a given inertial frame it will remain elastic with respect to any other inertial frame.

(c) In a head-on collision of two particles with masses m_1 and m_2, the initial velocities are $\mathbf{u_1}$ and $\mathbf{u_2} = \alpha\mathbf{u_1} (\alpha > 0)$ If the initial kinetic energies of the two particles are equal in the LAB frame, find the conditions on $\frac{u_1}{u_2}$ and $\frac{m_1}{m_2}$ so that m_1 will be at rest in the LAB frame after collision. Sketch how this collision will appear in the centre of mass frame.

4. **(Marks: 4 + 3 + 3)**

(a) Two particles with mass m_1 and m_2 interact with gravitational forces($\mathbf{F} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$) They start out from rest a distance ρ apart and are allowed to fall into each other. How long does it take for them to collide ?

(You may need to use the following result : $\int_0^1 \frac{dx}{\sqrt{\frac{1}{x}-1}} = \frac{\pi}{2}$)

(b) A particle is moving under the influence of an attractive central force given by $\mathbf{F}(\mathbf{r}) = -k\mathbf{r}$. Show that the radius vector $\mathbf{r}$ sweeps out equal areas in equal times.

(c) Assume Earth's orbit to be circular and that the Sun's mass suddenly decreases by half. What orbit does the Earth then have ? Will Earth escape the solar system ? Justify your answer.

5. **(Marks = 7 + 3)**

(a) A ball of mass M collides with a stick of uniform density with moment of inertia $I = \beta ml^2$ (relative to its centre, which is its centre of mass) The ball is initially travelling at a speed V_0 perpendicular to the stick. The ball strikes the stick at a distance d from the centre. The collision is elastic. Find the resulting translational and rotational speeds of the stick, and also the resulting speed of the ball.

(b) Give an example of a situation where the angular momentum $\mathbf{L}$ and the angular velocity ω do not point in the same direction. Demonstrate clearly in the example that the two vectors are not parallel.